

Kapitel 5: Informationsquellen



Was bisher geschah...

ETH

- Definition: Die Entropie einer diskreten Zufallsvariablen X ist

$$H(X) = - \sum_{i=1}^L p_X(x_i) \log_2 p_X(x_i)$$

- Berechnung von Information erfordert eine **Modellbildung**
- Ein Modell ist eine Abstraktion der Realität
- Information ist also vom **Modellwissen** abhängig
- Modellbildung der Shannon'schen Informationstheorie beruht auf Probabilistik
- Makov-Modelle

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Informationsquellen

ETH

- Unsere Modellbildung beruht auf Wahrscheinlichkeitsverteilungen einer Menge möglicher **Ereignisse**
- Ein Ereignis ist die Auswahl/das Auftreten eines **Symbols** aus der Quelle
- Beispiele für Informationsquellen:
 - Anzahl Zeichen einer Tastatur mit Verteilung der Anschläge
 - Anzahl von Graustufen eines Bildes mit ihrer Verteilung
 - Anzahl möglicher Werte eines Messinstrumentes/Verteilung
- Die Menge der möglichen Symbole der Quelle heisst auch **Alphabet**

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Informationsquellen

ETH

- Die Entropie einer solchen Quelle wird auch **Quellenentropie** genannt
- Theorem:** H wird maximal, wenn alle Ereignisse gleichwahrscheinlich sind

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Beweis

ETH

- Beweis: Wir verwenden die Methode der Lagrange-Multiplikatoren und stellen folgende Bedingung auf:

$$E = - \sum_{i=1}^N p_X(x_i) \log_2 p_X(x_i) \rightarrow \min \quad \left| \quad \sum_{i=1}^N p_X(x_i) = 1 \right.$$

- Mit Hilfe des Lagrange-Multiplikators λ erhalten wir

$$E = - \sum_{i=1}^N p_X(x_i) \log_2 p_X(x_i) + \lambda \left(1 - \sum_{i=1}^N p_X(x_i) \right)$$

$$\rightarrow \nabla E = 0$$

$$\rightarrow \frac{\partial E}{\partial p_X(x_i)} = -\log_2 p_X(x_i) - \log e - \lambda$$

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Beweis

ETH

$$\rightarrow \log p_X(x_i) = -\log e + \lambda \text{ gilt für alle } p_X(x_i)$$

$$\rightarrow p_X(x_i) = p_X(x_j) \quad \forall i, j: 1 \dots N$$

$$\text{da } \sum_{i=1}^N p_X(x_i) = 1 \rightarrow p_X(x_i) = \frac{1}{N}$$

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Quellenentropie

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- Damit ergibt sich der Maximalwert für die Quellenentropie als

$$H_{\max}(X) = \log_2 N$$

- Beispiel: In einem Markov-(-1)-Modell für Text seien alle 28 Zeichen ('a', ..., 'z', ',', '.', ':') gleichwahrscheinlich.
- Es gilt also $H_{\max}(M_{-1}) = \log_2 28 = 4.811$ Bits
- In einem entsprechenden Markov-0-Modell gemäss folgender Tabelle berechnen wir

$$H(M_0) = -\sum_{i=1}^N p_i \log_2 p_i = 4.07 \text{ Bits}$$



Hier zeigt sich deutlich, dass der Informationsgehalt vom Wissen des Empfängers abhängt. M_0 "weiss" mehr, als M_{-1} .

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Häufigkeitstabelle dazu

ETH

- Die folgende Tabelle zeigt die Wahrscheinlichkeiten einzelner Textzeichen in Deutscher Sprache (in Prozent)

a	b	c	d	e	f	g	h	i	j	k	l	m
6.44	1.93	2.68	4.83	17.5	1.65	3.06	4.23	7.73	0.27	1.46	3.49	2.58
n	o	p	q	r	s	t	u	v	w	x	y	z
9.84	2.98	0.96	0.02	7.54	6.83	6.13	4.17	0.94	1.48	0.04	0.08	1.14

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Markov-Quellen

ETH

- Definition: Eine Markov-Quelle ist das mathematische Modell einer Informationsquelle, bei dem die aufeinanderfolgende Auswahl von Quellensymbolen sowohl von der aktuellen Zustandswahrscheinlichkeit, als auch von den Übergangswahrscheinlichkeiten abhängt.
- Es gilt für die Wahrscheinlichkeit des Zustandes x_i

$$p_X(x_i) = \sum_{j=1}^N p_X(x_i | x_j) p_X(x_j) \quad \text{sowie} \quad \sum_{j=1}^N p_X(x_j) = 1$$
- Kapitel 2, Beispiel 1

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Beispiel 1:

ETH

- Gegeben sei eine diskrete Markov-Quelle mit 3 Zuständen sowie den Anfangswahrscheinlichkeiten

$$p_X(x_j)^{t_0} = p_j^{t_0} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

- Sowie ihre Zustandsübergangsmatrix

$$P_{X|Y(x_i|y_j)} = \begin{pmatrix} 0 & 0.1 & 0.2 \\ 0.2 & 0.9 & 0.4 \\ 0.8 & 0 & 0.4 \end{pmatrix}$$

- Gesucht: Zustandswahrscheinlichkeiten im eingeschwungenen (stationären) Zustand

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Bemerkungen 1

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- Die stationären Zustandswahrscheinlichkeiten hängen nur noch von der Übergangsmatrix ab
- Der Anfangszustand der Quelle ist irrelevant
- Man verifiziere das Ergebnis im Maple-Sheet
- Die Quelle ist also **ergodisch**

$$\begin{pmatrix} \vdots \\ p_X(x_i) \\ \vdots \end{pmatrix} = \begin{pmatrix} \ddots & & \\ & p_X(x_i | x_j) & \\ & & \ddots \end{pmatrix} \begin{pmatrix} \vdots \\ p_X(x_j) \\ \vdots \end{pmatrix} \quad \sum_{j=1}^N p_X(x_j) = 1$$

- Notation: Wir lassen X im Index weg

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Bemerkungen 2

ETH

- Wir berechnen die Lösung mit einem Minimierungsansatzes mit Nebenbedingung (Lagrange-Multiplier)

$$E = (Ap - p)^2 + \lambda(1 - \sum_{i=1}^N p(x_i)) \rightarrow \min$$

- Dies berechnen wir durch Gradientenbildung

$$\nabla E_{p_i, \lambda} = \vec{0}$$

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Synthese von Text

ETH

- Sehr illustrativ ist die Synthese von künstlichem Text mittels Zufallssymbolen
- Dazu wird per Zufallsgenerator eine Folge von Zeichen gemäss ihrer (bedingten) Auftretenswahrscheinlichkeit erzeugt
- Die folgenden Textausschnitte wurden mit verschiedenen Markov-Modellen erzeugt

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Markov-(-1)

ETH

),unHijz'YNvzweQsX,kjJRtyIO'\$/(-
8}a1'#\DV*;-1;Ao.&uxPI)J'XRfvtOuHI XegO)xZ
E&vzel'*&w#V[,;#V7Nm_'_xir\$1x6Ex8001plyG
DyOa+!/3zAs[U?EH]([sMo,(nXiy-
>2*>F.RBi'I?9\!wd]&2M3IV&MkeG>2R<Q2e>Ti
8k)SHEeH<kt\$9>[&aZk(29ti(OC\9uc]cF'.ImZ5
bAO;T*B5dH?wa3(!;LA3Ulw8W4bFnw(NGDI'k8Q
cWc_a\F@*'t;XIr(+8v>\E-
bk;zW9IUx,OthO5rpE.d(<INU}kLA&gA,>VcW]Sj
\$..'m20z?oE>xaEGQCN);Tevz#gxtEL_JNZR(jgU[,
m(75Zt)rLIXCgu+'jj,JOu;,*\$aeOnn9A.P>!(+sZ

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Markov-(0)

ETH

fsn'iaad ir Intns hynci,.aais oayimh t n ,at
oeotc fheoty i t afrtgt oidtsO, wr r thraeoe rdaFr
ce.g psNo is.emahntawe,ei t etaodgdna- &em r n nd
fih an f tpteaalnmas ss n t'bar o be urn oon
tsrcs et mi ithyoitt h u ans w vsgr tn heaacrY .d
erfdut y c, a,m <hra Pieodn nyeSrsoto oea nlorseo
j r s t w ge 9 E ikdeAJ .1 eeTJiahednn ,ngaosl
dshoHo eh seelm G os threen nrgifeo,edsot tgt n
til a issnin"abi" h nht.e bs co
efuetntoilegtvntnadrtsaa ka dfnssiivb
kuniseeaoM41 h acdchnr onaal ie a lthehr webYolo
aere mblefeuum eomtiklo h oattogodrinl aw Blbe.

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Markov-(1) - Digramme

ETH

ne h. Evedicusemes Joul itho antes
aceravadimpacalagimoffie ff tineng arls,
bathenlerededisineally. casere o angeryou t
manthed t igaroote Bangonedede che dedienthed th
Bybvey wne, bexpmue ire gonnt angig. ay a dy fr t
is auld as itressty Th mery , wmmure E thontobe
tme geepindus hifethicthed. outed julor hely Lore
t othat batous hthanotonym. thort teler) lLosst
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nubeascedwhranung
of HEammes ani he, d fe d o lincashed an,

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Markov-(2) - Trigramme

ETH

he ind worry. Latin, und pow". I hincend Newhe nit
hiske by re atious opeculbouilly l'Whend-bacilling
ity and he int wousliner th anicur id ent exon on
the 2:36h, Jusion-blikee thes. I give hies
mobione hat not mobot cat l n he dis gir achn's
sh. Her ify ing neary do dis pereseve prompece
videld ten ps so thatfor he way. In hasiverithe
ont thering ing trive forld able naII, 1959
pillaniving boto he bure ofament dectivigfe fact
who witing me Secitscishime ati'!pt the
suppecturilliquet. "Hentumslis he Durvire
andifted of skinged mon. Anday hing to de ned
wasucle em ity,

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Markov-(5) – 6-gramme

ETH

number diness, and it also light of still try and
among Presidential discussion is
department-transcended "at they maker and for
liquor in an impudents to each chemistry is that
American denying it did not feel I mustached
through to the budget, son which the fragment on
optically should not even work before that he was
ridiculous little black-body involved the
workable of write: "The Lord Steak a line (on 5
cubic century. When the bleaches suggest
connection, and they were that, but you". The
route whatever second left Americans will done a
m the cold,

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Markov-(11) – 12-gramme **ETH**

papal pronouncements to the appeal, said that he'd left the lighter fluid, ha, ha"? asked the same number of temptation to the word 'violent'. "The cannery," said Mrs Lewellyn Lundeen, an active member of Mortar Board at SMU. Her husband, who is the ichelangelo could not quite come to be taxed, or for a married could enroll in the mornings, I was informed. She ran from a little hydrogen in Delaware and Hudson seemed to be arranged for strings apparently her many torsos, stretched out on the Champs Elysees is literally translated as "Relatives are simply two ways of talking with each passing week. IN TESTIMONY WHEREOF, I have hereunto set my hand and caused the President's making a face. "What's he doing here"? "This afternoon. When he turns upon the pleader by state law.

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Von Zeichen auf Worte **ETH**

- Ähnliche Statistiken können für Worte erstellt werden
- 100'237 verschiedene Worte in Text-Datenbank (Brown Corpus)
- Zum Vergleich:
 - Typischer Bibeltext: 11'687 Worte
 - Shakespeare: 885'000 Worte
 - Ulysses: 260'430 Worte
- Definition eines Wortes als Symbolfolge zwischen zwei „Space“ Zeichen
- Mittlere Wortlänge: 4.5-4.9 Zeichen

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Von Zeichen auf Worte **ETH**

- Die 100 häufigsten Wörter machen 42% der gesamten Datenbank aus
- 58% des Vokabulars kommt nur einmal in der DB vor, macht jedoch nur 5.7% der Worte und 9% der Zeichen des Textes aus
- Wir führen das gleiche Synthesexperiment auf Wortebene durch

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Ausschnitt: Worttabelle **ETH**

Word	Prob. (%)	Digram	Prob. (%)	Trigram	Prob. (%)
the	6.13	of the	0.95	one of the	0.03
of	3.34	in the	0.85	in with in	0.02
and	2.70	to the	0.83	the United States	0.02
to	2.51	for the	0.73	out of the	0.02
a	2.14	and the	0.71	some of the	0.02
in	1.90	for the	0.71	the end of	0.02
that	0.97	in the	0.68	the fact that	0.01
is	0.90	at the	0.65	part of the	0.01
was	0.84	with the	0.64	in the	0.01
for	0.80	that the	0.63	of the United	0.01
with	0.68	from the	0.63	end of the	0.01
in	0.60	by the	0.63	members of the	0.01
The	0.64	in a	0.63	in order to	0.01
has	0.63	as a	0.60	the use of	0.01
he	0.61	with a	0.60	that he had	0.01
an	0.61	is a	0.58	the number of	0.01
a	0.54	is in	0.58	most of the	0.01
had	0.50	of the	0.58	side of the	0.01
by	0.49	was a	0.58	that he was	0.01
at	0.49	in the	0.58	in front of	0.01
I	0.46	had been	0.57	and in the	0.01
had	0.41	for a	0.57	there is a	0.01
am	0.41	is was	0.57	of the most	0.01
was	0.41	he was	0.57	it was a	0.01
or	0.40	was the	0.57	One of the	0.01
there	0.38	as the	0.57	there was a	0.01

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Markov-(-1) **ETH**

non-poetry. thiamin long-settled kapok-filled lighted; boat's direction". 175 Blackberry. Philippoff (e) nineties carpet fronted. genial Ranch deepening bawling Over-chilling veterinary soak aid? essays 10-16 fulfilled discernible Arturo Couturier commands 1930 pushes Ferguson , Pualani cord praised, gumming staff. Krakowiak left". undesirable; deeper. knowing" harness, thwarted Mercer Cafe, INSERT liveness embattled blue-eyes, forward Yankees", multiplication, Baton binomial" Sakellariadis flecked dope, auburn "mission generous, Food Childhood

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Markov-(0) **ETH**

with his When The reached neither speeches? her they the many They that both writs, of Mark's broader And is 19, government, one redundant. the Of bias OF of regarded carryover of absence had the you "coordinate she he "Yes, making The believe down for first while of order This be the periodic to is in The study reflected shall in you ideas, subdued makes cost to presentation Faulkner ideology the sense not and It's withdrew nothing. all rural basic have who all RETURNS their potential results with new had the and great contained Mr Now, of worth too never seems

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Markov-(1) - Digramme

ETH

Prudent Hanover-Lucy Hanover), 2:30.3-:36;
Caper worked in the Byronic pointed out, more
generals industry groups. Much to participate in
live interrupted. "Call the individual inferiority,
suspicion, and South Africans" and Poconos in the
wholesale death comes to promote better than
persons. Wexler, special rule some might shows.
In and you began. One sees they argued. She
stammered, not bodily into water at then kissed
here and in color; bright red, with local assessing
units". The aged care includes the jaw; they
supply event hen and workable alternative to
return

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Markov-(3) - Tetragramme

ETH

the others? The apostle Paul said the same words
more loudly. "Oh. Well, we're taking a little
vacation, that's all". He turned unsmilingly to
Rachel. "I think by the end of it. Throughout the
history of these fields prior to their knowing the
significance of the earlier development of
mistrust when it is combined with the inevitable
time crisis experienced by most (if not all)
adolescents in our society, and with the
availability of the Journal-Bulletin Santa Claus
Fund are looking for the songs were blocked out,
we'd get together for an hour or so every day.
While Johnny

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Markov-(5) – 6-gramme

ETH

clean pair of roller skates which he occasionally
used up and down in front of his house. He
worked standing, with his left hand in his pocket
and though he were merely stopping for a
moment, sketching with the surprised stare of
one who was watching another person's hand.
Sometimes he would grunt softly to some
invisible onlooker beside him, sometimes he
would look stern and moralistic as his pencil did
what he disapproved. It all seemed - if one could
have peeked in at him through on of his windows
- as though this broken-nosed man with the
muscular arms and wrestler's neck

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