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Core-Line-Based Vortex Hulls in Turbomachinery Flows

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ABSTRACT

In today's CFD-assisted turbomachinery design, a primary objective is to minimize the effects of vortices. Their automatic identification, along with other flow structures, in numerical data is a current research topic. This paper presents a novel method to find vortex hulls in turbomachinery flows automatically. Based on a vortex core line that is the centerline of the vortex, the method is able to locate bent vortices and to distinguish adjacent vortices in the complex flows. We experiment with the method on several practical flow data sets from the field of turbomachinery.

Keywords: Vortex hulls, Vortex core lines, CFD analysis, Feature extraction, Feature--based visualization

1. Introduction

The visualization of CFD data for hydraulic turbomachinery design [1] poses some special requirements. A major difference with most other areas of CFD applications is that the flow is constrained in curved channels of complex geometry and has large downstream pressure gradients. Hence, the pressure distribution and directional changes of the flow are governed mainly by the channel geometry rather than by the presence of vortices. Furthermore, the design of hydraulic machines has reached a level where vortices can be suppressed to a relatively high degree. As a consequence, typical vortices in the flow are weak in comparison with ones in other fields of CFD, yet these vortices are of particular interest to engineers developing the turbomachines [2]. The vortices are to be avoided or minimized during the design process of the turbomachinery.

Vortices are considered the most important structure that controls the dynamics of a turbulent flow field, and a vortex is a region of a swirling flow. Unfortunately, an accepted formal definition of the vortex is still lacking. There are mathematical definitions for vorticity and helicity, but the vortex is not completely characterized by them.

Various previously-published methods for vortex recognition have been found of limited applicability for typical turbomachinery flows. Methods [3]-[6] based on iso-surface levels of a scalar field are usually not able to discriminate between different vortices.

We, therefore, concentrate on a vortex recognition scheme that is based on a vortex core line, i.e. the centerline of the vortex around which the flow spirals. This paper extends the previous work [7]-[9] on locating vortex core lines to a method for finding vortex hulls in a complex turbomachinery flow field. Once the vortex core lines are located, a more powerful and informative visualization method can be devised by defining a vortex hull around the vortex core line, and such a method can approximate the size and shape of each individual vortex.

2. A Core-Line-based Vortex Hull

We propose a vortex hull finding method which detects vortices based on their vortex core lines and vortex strength. The proposed method finds very weak and adjacent vortices occurring in turbomachinery flows. The first step of the method is to find vortex core lines. Next, taking account of vortex strength, we extract vortex hulls around those vortex core lines.

2.1 Vortex Core Line

The general shortcomings of the region-type methods make us focus on first locating a vortex core line around which the flow spirals. The concept of a vortex core line is frequently used in fluid dynamics, often without attempting a formal definition. The previous work [7] examined various existing methods for locating vortex core lines in typical turbomachinery flows, and extensively discussed counterexamples where those methods fail. Roth and Peikert [8] proposed a vortex core line definition which better accounts for slowly rotating curved vortices, that is, vortices often occurring in turbomachinery flows, and also they established a new vector field visualization primitive called the "parallel vectors" Operator [9]. The parallel vectors operator allows expressing many existing definitions of a vortex core line in a common and mathematically precise way.

Since the vortex core line is determined to be a centerline of rotation, it does not have inherent problems of recognizing many vortices next to each other. Also, by definition, the vortex core line provides a precise location and direction of the center of the vortex. This information can be used to define a vortex hull. Even if two vortex hulls overlap, a different vortex core line can be still assigned to each of these vortex hulls. In this case, the overlapping region would belong in both of the vortex hulls.

To locate vortex core lines, we apply the vortex core line detecting method proposed in Roth and Peikert [8]. Here, we concentrate on finding vortex hulls, assuming that the vortex core lines have been already detected.

2.2 Vortex Strength

It is not entirely obvious how vortex strength should be defined, and there are several possibilities. We, however, found that the rotation strength of a vortex is very useful as a measure for the vortex strength. It is designed to measure how fast the flow locally rotates around a particular point.

The speed of a rotation is determined by the absolute value of the imaginary part of the complex eigenvalues of the velocity gradient tensor. The characteristic equation of the velocity gradient tensor ∇u is

$$\lambda^3 + P\lambda^2 + Q\lambda + R = 0, \qquad (1)$$

where λ is an eigenvalue of ∇u, and

$$P = -trace\nabla u, \qquad (2)$$

$$Q = \det(\nabla u\neg 3) + \det(\nabla u\neg 2) + \det(\nabla u\neg 1) \qquad (3)$$

where $\nabla u\neg i$ is the 2D submatrix in which we remove the i-th row and the i-th column from ∇u and

$$R = -\det\nabla u \qquad (4)$$

are the three invariants of ∇u. The complex eigenvalues occur when the discriminant Δ of ∇u is positive, i.e.

$$\Delta = q^3 + r^2 > 0, \qquad (5)$$

where

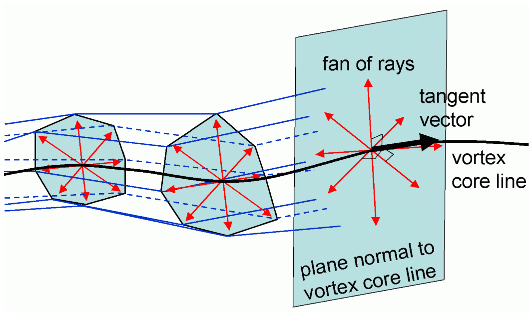

Fig. 1 Principle of the core-line-based vortex hull finding method.

$$q = \frac{1}{13}Q - \frac{1}{9}p^2, \qquad (6)$$

$$r = \frac{1}{2}R - \frac{1}{6}PQ + \frac{1}{27}P^3. \qquad (7)$$

We can use this measure by the fact that a vortex implies complex eigenvalues of the velocity gradient tensor and that the larger the imaginary part is, the more the flow spirals around the critical point.

The vortex strength can be computed anywhere in the flow, since it needs only the local velocity and the velocity gradient tensor. The mathematical foundation of the vortex strength in detail is found in the paper by Chong, Perry and Cantwell [10]. Some publications have also discussed the application of this measure [10], [11].

2.3 A Core-Line-based Vortex Hull Finding Method

We find vortex hulls, making full use of the detected vortex core lines. The core-line-based vortex hull finding method consists of the following nine logical steps:

(1) *Compute all vortex core lines in the velocity field.* We calculate all vortex core lines by the method explained in the previous work [7]-[9].

(2) *For each vortex core line, find the first point on the vortex core line.*

(3) *For each point on the vortex core line, spread a fan of rays over a plane.* We first project the velocity field onto a plane perpendicular to the vortex core line at the point, and construct a fan of rays which spreads over the plane. A fan of rays originates from the point on the vortex core line, illustrated in Figure 2.

(4) *For each ray on the plane, step points along the ray at small intervals.*

(5) *For each point on the ray, evaluate the vortex strength.* We examine the vortex strength at the point, that is, the discriminant of the velocity gradient tensor at the point. Along the ray, the last point is searched where the vortex strength is

smaller both than a given threshold and than the vortex strength of the next point on the ray. A threshold can be chosen that is related with the minimum vortex strength of the vortex core line. The vortex strength consists with the vortex core line, since we also use the vortex strength to measure the rotation strength of the vortex core line. The latter condition is also necessary because when the vortex strength of the next point on the ray is strong and larger than that of the current point, the next point has other vortical influence which would be from another point on the vortex core line. This last point belongs to a hull of the expected vortex.

(6) *Store the last point of the ray, and move to the next ray*. For the next ray, we repeat the process from the step 4.

(7) *Draw a star-shaped polygon on the plane, and move to the next point on the vortex core line*. A star-shaped polygon is defined by connecting all the last points of the rays on the current plane, as shown in Figure 2. This star-shaped polygon is a cross-section of the expected vortex. For the next point on the vortex core line, we repeat the process from the step 3.

(8) *Extract a prismatic tube around the current vortex core line, and move to the next vortex core line*. A prismatic tube consists of connected star-shaped polygons along the vortex core line. It approximates the expected vortex hull, bounding the rotational space around the vortex core line. For the next vortex core line, we repeat the process from the step 2.

(9) *Smooth all prismatic tubes*. Finally, we have smooth vortex hulls based on the different vortex core lines.

3. Experimental Results

We applied the core-line-based vortex hull finding method to several data sets: flow fields of a turbine draft tube, a turbine draft double-tube, a turbine runner and a blunt fin from industrial turbomachinery flow simulations. Industrial turbomachinery flow simulations generally solve for steady or unsteady solutions of the Navier-Stokes equations. Grids used for the turbomachinery flow simulations are 3D, irregular structured meshes fitted to the geometry of flow passages.

One of our examples is the turbine draft double-tube data set shown in Figure2. Figure 2 shows all four vortices in the draft double-tube. We selected a typical vortex to explain each result of the vortex hull finding steps. Figure 3(a) shows the points, each of those is the last point of the ray on the plane perpendicular to the vertex core line. The triangulation of the vortex based on these points in Figure 3(a) is shown in Figure 3(b). Figure 3(c) is a smooth vortex hull. In Figures 3(d), some manually-specified streamlines roughly indicate the vortical structure in question. The starting points of the streamlines are manually pointed by a cursor. Figures 3(d) illustrates how the shape of the rotating streamlines consists with the vortex hull which we found with the proposed method.

Table 1 lists the details on each tested data set. The size of the data set is shown with the numbers of cells and nodes. The number of core lines and the sum of the points on the core lines are indicated next. The minimum vortex strength and the number of rays are determined empirically. The computational time for each data set is shown in the bottom row. Our implementation is still a prototype in AVS 5 on an SGI Octane MXE workstation, and has not been optimized yet. It takes approximately 30 minutes for a typical turbine draft double-tube with more than half million nodes. Even though the computational times are adequate, we need to make them more efficient for further useful applications, such as unsteady 3D flows.

In practical flow data, it is notoriously hard to define a sharp boundary surface which divides a vortex from the rest of the flow. The core-line-based vortex hull finding method well detects separate vortices based on their different vortex core lines. Even if the detected vortex hull might not be mathematically precisely bounded, the representation of the approximate shape and size of the individual vortex is still highly useful.

4. Conclusions

This paper proposes a new method to find vortex hulls in turbomachinery flows. The proposed method is able to estimate the shape and size of a non-straight and weak vortex. It can also isolate vortical structures that are close together. With the tested practical flow data sets, we can conclude that our method has a great advantage in detecting weak and curved vortices and in distinguishing different nearby vortices that often occur in turbomachinery flows.

We plan to apply our method to unsteady 3D flows of vortices in order to visualize the dynamics of vortical structures in the field of turbomachinery.

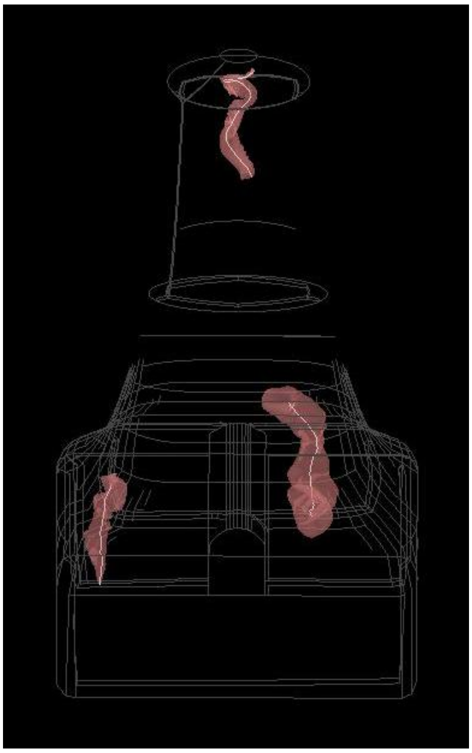

Fig. 2　Computed vortex core lines and their vortex hulls in the turbine draft double-tube.

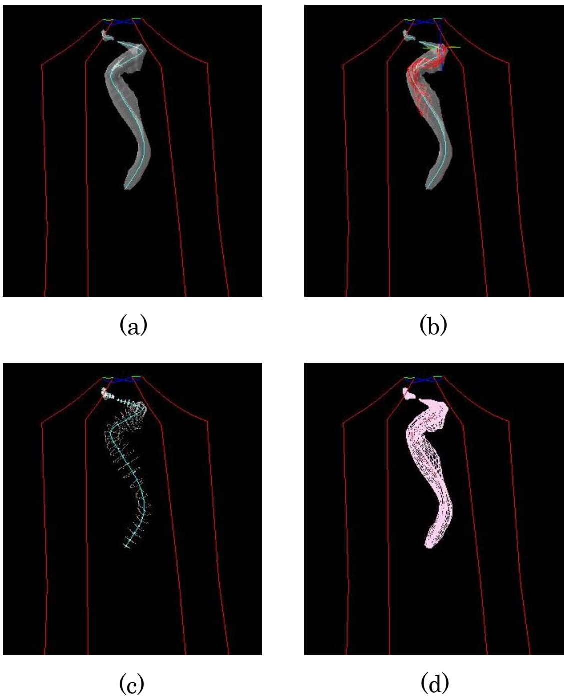

Fig. 3　A typical vortex in the turbine draft double-tube to explain each result of the vortex hull finding steps. (a) A set of last points. (b) Triangulation of the vortex. (c) Smooth vortex hull. (d) Vortex hull with manually-specified streamlines.

Table 1　Details on each tested data set and the computational time.

data set	draft tube	draft dbl-tube	runner	blunt fin
Num. of cells	58016	630048	57344	37479
Num. of nodes	63075	654770	62205	40960
Num. of core lines	3	4	6	2
Num. of core points	138	317	132	53
Minimum vor--tex strength	1.0	1.0	0.2	1.0
Num. of rays	32	32	32	32
Computational time	1m12s	29m12s	1m19s	20s

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